Let P be the point $(-3, 4, -1)$, R be the point $(1, 2, -7)$, and $\overrightarrow{PQ}$ be the vector $-3\vec{i} - 2\vec{j} + \vec{k}$. **SCORE: ____ / 103 PTS**

3 EACH EXCEPT AS NOTED

[a] If $\overrightarrow{PR}$ is parallel to $< -3, 2-c, b+1 >$, find the value of b.

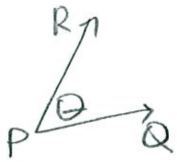
$\overrightarrow{PR} = \langle 4, -2, -6 \rangle = k\langle -3, 2-c, b+1 \rangle$ 4

$= \langle -3k, (2-c)k, (b+1)k \rangle$

$4 = -3k \qquad -6 = (b+1)k$

$k = -\frac{4}{3} \qquad -6 = -\frac{4}{3}(b+1) \longrightarrow b = \frac{7}{2}$

[b] Find $\angle RPQ$.



$\cos^{-1} \dfrac{\overrightarrow{PQ} \cdot \overrightarrow{PR}}{\|\overrightarrow{PQ}\| \|\overrightarrow{PR}\|} = \cos^{-1} \dfrac{-12+4-6}{(\sqrt{14})(2\sqrt{14})} = \cos^{-1} \dfrac{-14}{28}$

$= \cos^{-1} -\frac{1}{2}$

$= \frac{2\pi}{3}$ OR $120°$

[c] Find the area of triangle PQR.

$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3 & -2 & 1 \\ 4 & -2 & -6 \end{vmatrix} \begin{matrix} \vec{i} & \vec{j} \\ -3 & -2 \\ 4 & -2 \end{matrix} = \langle 14, -14, 14 \rangle$ 6

CHECK: $\langle 14, -14, 14 \rangle \cdot \langle -3, -2, 1 \rangle$
$= -42 + 28 + 14 = 0$
$\langle 14, -14, 14 \rangle \cdot \langle 4, -2, -6 \rangle$
$= 56 + 28 - 84 = 0$ ✓

$\frac{1}{2} \|\langle 14, -14, 14 \rangle\| = \frac{1}{2} |14| \, \|\langle 1, -1, 1 \rangle\|$

$= \frac{1}{2} \cdot 14 \cdot \sqrt{3}$

$= 7\sqrt{3}$

[d] Find the coordinates of Q.

$\langle x+3, y-4, z+1 \rangle = \langle -3, -2, 1 \rangle$

$x + 3 = -3$
$y - 4 = -2$
$z + 1 = 1$

$(x, y, z) = (-6, 2, 0)$

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[e] Find the general **(NOT point-normal)** equation of the plane which contains P, Q and R.

$\vec{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \langle 14, -14, 14 \rangle$ or $\langle 1, -1, 1 \rangle$, 6

$(x+3) - (y-4) + (z+1) = 0$

$x - y + z + 8 = 0$

+3 SIMPLIFY

[f] Find parametric equations of the line which is perpendicular to the plane in part [e], and also contains Q.

$\vec{d} = \vec{n}$

$x = -6 + t$
$y = 2 - t$
$z = t$

6

[g] Find symmetric equations of the line which is parallel to the line in part [f], and also is perpendicular to the plane in part [e], and also contains P.

$\vec{d}_2 = \vec{d} = \vec{n}$

$\frac{x+3}{1} = \frac{y-4}{-1} = \frac{z+1}{1}$, 6

$x + 3 = 4 - y = z + 1$

[h] Find a vector of magnitude 6 perpendicular to both $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

$\frac{6}{\|\overrightarrow{PQ} \times \overrightarrow{PR}\|}(\overrightarrow{PQ} \times \overrightarrow{PR}) = \frac{6}{14\sqrt{3}} \langle 14, -14, 14 \rangle$

$= 2\sqrt{3} \langle 1, -1, 1 \rangle$

$= \langle 2\sqrt{3}, -2\sqrt{3}, 2\sqrt{3} \rangle$ or $\langle -2\sqrt{3}, 2\sqrt{3}, -2\sqrt{3} \rangle$

Find all octants in which $xz < 0$ and $y < 0$ simultaneously. **SCORE: ____ / 10 PTS**

2 $x > 0, y < 0, z < 0 \rightarrow O_{4+4}$ or O_8, $3\frac{1}{2}$

2 $x < 0, y < 0, z > 0 \rightarrow O_3$, $2\frac{1}{2}$

Consider the sphere $x^2 + y^2 + z^2 + 14x + 8y - 12z + 65 = 0$. **SCORE: ____ / 15 PTS**

[a] Find the equation of the yz – trace. Describe the yz – trace.

$$x^2+14x+49+y^2+8y+16+z^2-12z+36=-65+49+16+36$$

$$(x+7)^2+(y+4)^2+(z-6)^2=36$$

$1\frac{1}{2}$ $x=0 \rightarrow 49+(y+4)^2+(z-6)^2=36$

$$(y+4)^2+(z-6)^2=-13$$

NO TRACE

2 EACH EXCEPT AS NOTED

[b] Find the equation of the xy – trace. Describe the xy – trace.

$1\frac{1}{2}$ $z=0 \rightarrow (x+7)^2+(y+4)^2+36=36$

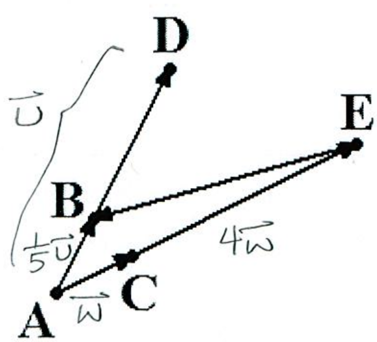
$$(x+7)^2+(y+4)^2=0$$

POINT $(-7,-4,0)$

In the diagram below, ABD and ACE are both line segments. **SCORE: ____ / 10 PTS**

CE is four times the length of AC, and AD is five times the length of AB. **(NOTE: The diagram is <u>NOT</u> drawn to scale.)**

If $\vec{u} = \overrightarrow{AD}$ and $\vec{w} = \overrightarrow{AC}$, find an expression for $\overrightarrow{EB}$ in terms of $\vec{u}$ and $\vec{w}$.



$$\overrightarrow{EB} = \overrightarrow{EA} + \overrightarrow{AB}$$

$$= \underbrace{-5\vec{w}}_{5} + \underbrace{\tfrac{1}{5}\vec{u}}_{5} \text{ OR } \tfrac{1}{5}\vec{u} - 5\vec{w}$$

Fill in the blanks. List all correct answers. **SCORE: ____ / 12 PTS**

[a] If $\vec{u} \cdot \vec{u} = 6$, then $\vec{u} \times \vec{u} =$ $\vec{0}$ and $\|\vec{u}\| =$ $\sqrt{6}$.

$\langle 2,0,-3\rangle = \langle 4-x,-1-y,-8-z\rangle$

[b] If the terminal point of $\vec{v} = 2\vec{i} - 3\vec{k}$ is $(4, -1, -8)$, then the initial point of $\vec{v}$ is $(2,-1,-5)$.

2 EACH

[c] The equation of the x – axis is $y=z=0$ and the equation of the xz – plane is $y=0$.

[d] If you start at the point $(-1, -5, 1)$, then move 3 units to the right, 8 units downward and 6 units backward,

you will be at the point $(-7,-2,-7)$. $(-1-6,-5+3,1-8)$